Indeterministic Quantum Gravity and Cosmology

IX. Nonreality of Many-Place Gravitational Autolocalization: Why a Ball Is Not Located in Different Places at Once

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Abstract

This paper is a sequel of papers [1-8], being an immediate continuation and supplement to the last of them, where gravitational autolocalization of a body has been considered. A resulting solution, which describes a one-place location, has been called gravilon. Here it is shown that a gravilon is the only solution, i.e., that many-place gravitational autolocalization is unreal. This is closely related to nonreality of tunneling in the conditions under consideration.

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Why, then, do we not experience macroscopic bodies, say cricket balls, or even people, having two completely different locations at once? This is a profound question, and present-day quantum theory does not really provide us with a satisfactory answer.

Roger Penrose

Introduction

In the previous paper [8] of this series, the possibility of a gravitational autolocalization has been established. A resulting solution describes a one-place location of a body, which is called gravilon. But in [8] the question on nonreality of many-place gravitational autolocalization has not been even raised. This paper makes up for this essential deficiency.

The answer to the question is positive: Many-place gravitational autolocalization is unreal. The essence of the matter is as follows.

For the sake of simplicity, we consider the case of a two-place location. A wave function of the center of mass of a ball in such a situation depends on some parameters: the distance between location places and coefficients in the linear combination of one-place functions. The requirement that the wave function be an eigenfunction of the Hamiltonian results in a relation for the parameters, which singles out a set of measure zero for the admissible values of the parameters. This result is obtained neglecting the tunneling between the locations, so that nonreality of many-place location is closely related to nonreality of tunneling in the conditions under consideration.

Thus the only solution to the problem of gravitational autolocalization is a one-place location, i.e., a gravilon.

1 Basic equation

Let $\psi(\vec{r})$ be a wave function of the center of mass of a ball with a mass M [8]. ψ must satisfy the Schrödinger equation,

$$H\psi = \epsilon\psi \tag{1.1}$$

where the Hamiltonian

$$H \equiv H[\psi] = -\frac{\hbar^2}{2M}\triangle + V(\vec{r};\psi) \tag{1.2}$$

and V is the potential energy of the ball in a gravitational field caused by the ball itself with the wave function ψ.

2 Two-place function

For the sake of simplicity, let us consider the case of a two-place location:

$$\psi \equiv \psi(\vec{r}) = c_{\mathrm{I}}\psi_{\mathrm{I}}(\vec{r}) + c_{\mathrm{II}}\psi_{\mathrm{II}}(\vec{r}) \tag{2.1}$$

where

$$\psi_{\rm I} = \psi_1(\vec{r}), \quad \psi_{\rm II} = \psi_2(\vec{r} - \vec{R}) \tag{2.2}$$

and

$$R \gg a_0,\ r_{0i}, \qquad i = 1, 2, \tag{2.3}$$

here a_0 is the radius of the ball and r_{0i} is an effective radius of ψ_i. We may put

$$c_{\rm I} = e^{{\rm i}\alpha}\sqrt{1-b^2}, \qquad c_{\rm II} = e^{{\rm i}\alpha}e^{{\rm i}\beta}b, \qquad 0 \le b \le 1, \tag{2.4}$$

so that, dropping $e^{{\rm i}\alpha}$, we have

$$\psi \equiv \psi(\vec{r}; b, \beta, \vec{R}) = \sqrt{1-b^2}\psi_1(\vec{r}) + e^{{\rm i}\beta}b\psi_2(\vec{r} - \vec{R}). \tag{2.5}$$

Now, in view of eq.(2.3), we neglect the tunneling between the two locations and choose ψ_1, ψ_2 to be eigenfunctions of the Hamiltonian,

$$H\psi_i = \epsilon_i\psi_i. \tag{2.6}$$

We have

$$H = H[b, \beta, \vec{R}],\ \psi_1 = \psi_1(\vec{r}; b, \beta, \vec{R}),\ \psi_2 = \psi_2(\vec{r} - \vec{R}; b, \beta, \vec{R}),\ \epsilon_i = \epsilon_i(b, \beta, \vec{R}). \tag{2.7}$$

3 The set of two-place states: measure zero

We have from eqs.(2.5), (2.6)

$$H\psi = \sqrt{1-b^2}\epsilon_1\psi_1 + e^{{\rm i}\beta}b\epsilon_2\psi_2, \tag{3.1}$$

so that for (1.1) to be satisfied, the relation

$$\epsilon_1(b, \beta, \vec{R}) = \epsilon_2(b, \beta, \vec{R}) \equiv \epsilon(b, \beta, \vec{R}) \tag{3.2}$$

must be fulfilled.

Eq.(3.2) determines a set of measure zero for admissible values of the parameters b, β, and $\vec{R}$. This implies that many-place gravitational autolocalization is unreal.

4 Tunneling

Let us consider the tunneling, which was neglected. The condition of this neglect is

$$DV \ll \Delta\epsilon \tag{4.1}$$

where

$$D \sim e^{-2\sqrt{2M(V-\epsilon)/\hbar^2}L}, \tag{4.2}$$

is the transmission coefficient and $\Delta\epsilon$ is the distance between energy levels.

We have [8]

$$V - \epsilon \approx V \sim \frac{\kappa M^2}{a}, \qquad a = a_0 + r_0, \tag{4.3}$$

where κ is the gravitational constant,

$$\Delta\epsilon \sim \frac{\hbar^2}{Ma^2}, \tag{4.4}$$

and

$$L = R. \tag{4.5}$$

Thus eq.(4.1) reduces to

$$\lambda e^{-\sqrt{\lambda}R/a} \ll 1 \tag{4.6}$$

with

$$\lambda \sim \frac{\kappa M^3}{\hbar^2} a, \tag{4.7}$$

or

$$\lambda \sim \left(\frac{M}{m_P}\right)^3 \frac{a}{l_P}, \tag{4.8}$$

where m_P is the Planck mass and l_P is the Planck length.

We have by eq.(2.3)

$$\frac{R}{a} \gg 1, \tag{4.9}$$

so that the condition (4.6) is fulfilled. In fact, since by [8]

$$\lambda \gg 1 \tag{4.10}$$

holds, the inequality (4.6) is very strong.

Thus nonreality of many-place gravitational autolocalization is closely related to nonreality of tunneling in the conditions under consideration.

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References

[1] Vladimir S. Mashkevich, *Indeterministic Quantum Gravity* (gr-qc/9409010, 1994).

[2] Vladimir S. Mashkevich, *Indeterministic Quantum Gravity II. Refinements and Developments* (gr-qc/9505034, 1995).

[3] Vladimir S. Mashkevich, *Indeterministic Quantum Gravity III. Gravidynamics versus Geometrodynamics: Revision of the Einstein Equation* (gr-qc/9603022, 1996).

[4] Vladimir S. Mashkevich, *Indeterministic Quantum Gravity IV. The Cosmic-length Universe and the Problem of the Missing Dark Matter* (gr-qc/9609035, 1996).

[5] Vladimir S. Mashkevich, *Indeterministic Quantum Gravity V. Dynamics and Arrow of Time* (gr-qc/9609046, 1996).

[6] Vladimir S. Mashkevich, *Indeterministic Quantum Gravity and Cosmology VI. Predynamical Geometry of Spacetime Manifold, Supplementary Conditions for Metric, and CPT* (gr-qc/9704033, 1997).

[7] Vladimir S. Mashkevich, *Indeterministic Quantum Gravity and Cosmology VII. Dynamical Passage through Singularities: Black Hole and Naked Singularity, Big Crunch and Big Bang* (gr-qc/9704038, 1997).

[8] Vladimir S. Mashkevich, *Indeterministic Quantum Gravity and Cosmology VIII. Gravilon: Gravitational Autolocalization* (gr-qc/9708014, 1997).